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A concise way to see both results is as follows:

1. Length of the body diagonal

A rectangular prism with edges 4.0cm, 2.0cm, and 3.0cm has a body (space) diagonal of length

$$\sqrt{4.0^2 + 2.0^2 + 3.0^2} = \sqrt{16 + 4 + 9} = \sqrt{29} \approx 5.4\text{cm (to one decimal place)}.$$

2. Measure of $\angle AFH$

The angle $\angle AFH$ is formed at F by the **body diagonal** \overrightarrow{FA} and the **base diagonal** \overrightarrow{FH} . In a convenient coordinate setup (taking F as the origin),

- $F = (0, 0, 0)$,
- $H = (4, 2, 0)$ (diagonally opposite F in the 4cm \times 2cm base),
- $A = (4, 2, 3)$ (directly above H , giving the full 3cm height).

Then

$$\overrightarrow{FA} = (4, 2, 3), \quad \overrightarrow{FH} = (4, 2, 0).$$

Their dot product is

$$\overrightarrow{FA} \cdot \overrightarrow{FH} = (4)(4) + (2)(2) + (3)(0) = 16 + 4 + 0 = 20,$$

and their magnitudes are

$$\|FA\| = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{29}, \quad \|FH\| = \sqrt{4^2 + 2^2 + 0^2} = \sqrt{20}.$$

Hence

$$\cos(\angle AFH) = \frac{\overrightarrow{FA} \cdot \overrightarrow{FH}}{\|FA\| \|FH\|} = \frac{20}{\sqrt{29}\sqrt{20}} = \frac{20}{\sqrt{580}} \approx 0.8305.$$

Taking the inverse cosine (in degrees) gives

$$\angle AFH \approx 34.0^\circ \text{ (to one decimal place).}$$

Answer:

- $AF \approx 5.4\text{cm}$
- $\angle AFH \approx 34.0^\circ$

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